## THE DISCUSSION

Continues

Excerpts from Sid Soni's original letter (the complete text of which appeared in MT March 2012), several responses to the original letter, and Soni's reply to those responses follow.

## THE CASE FOR REPEATING ALGEBRA 1

At midyear in my algebra 2 course, I distributed my students' grades, which ranged from the 40 s to the 90 s. I felt that the failing students had simply not mastered the prerequisite algebra skills required to perform in this course, ... so I administered an informal diagnostic to see just how closely my students' first-year algebra skills correlated with their second-year algebra performance....

The correlation between the students' scores on the algebra skills diagnostic and their algebra 2 midyear average was $r=.75$. Further, the bottom 20th percentile (those students scoring $44 \%$ or less) correlated to a $100 \%$ fail rate in the algebra 2 course at midyear. This was strong evidence that this subset of the class simply did not have the prerequisite algebra skills to pass algebra 2. What steps can be taken to avoid this situation in the future?
... Courses with axiomatic prerequisite skills should require students to pass a basic entrance exam....

The ostensible choice is either to promote the student into a class he or she probably cannot pass or to give him or her another chance to master the basics under different circumstances-having a different teacher, being one year older, seeing the material again, and so on. For students who cannot demonstrate algebra skills, I believe that repeating first-year algebra is the best recourse....

One rationale for promoting borderline students is to ensure that they complete four years of high school mathematics, thus bolstering their college applications....

Weak students, despite having taken four years of mathematics, will not likely qualify for the caliber of college that encourages doing so.

Further, at many colleges, particularly less competitive ones, incoming students are given a placement exam that is heavily skewed toward measuring their algebraic skills....

If I had to wager on the outcome of this exam, I would predict that this cohort will be retaking algebra in college, thus begging the question: Are these students better off repeating algebra in grade 10 or in grade $13 ?$

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We appreciate the interest and value the views of those who write. Readers commenting on articles are encouraged to send copies of their correspondence to the authors. For publication: All letters for publication are acknowledged, but because of the large number submitted, we do not send letters of acceptance or rejection. Letters to be considered for publication should be in MS Word document format and sent to $\mathbf{m t}$ @nctm.org. Letters should not exceed 250 words and are subject to abridgment. At the end of the letter include your name and affiliation, if any, including e-mail address, per the style of the section.

## REACTIONS FROM READERS

"I agree completely. I teach algebra 2 and have seen exactly what you describe, although many of my students are even less prepared. Not only have they no concept of algebra 1 skills, especially factoring, but also many of my juniors and seniors are completely lacking in basic arithmetic
facts and have no number sense at all. They need to return to fourth grade and repeat all mathematics from there.

Thank you for a great letter. I am going to share it with my colleagues.

Glen Cooke
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Cookeville, TN, Mar. 2, 2012

1)I agree with the gist of the letter. I teach at a vocational high school, and we struggle with the overall low and inconsistent algebra readiness skills of our rising ninth graders. We are currently revamping our four-year mathematics paths to emphasize college readiness for all our students. All entering students will take algebra 1 , even if they had it in middle school. Because our students have a wide array of mathematics skills and backgrounds, we will offer two levels of the coursebasic and advanced—and will differentiate within each. Ninth graders who are designated as Title I students will also take a separate mathematics lab class that will provide remedial instruction in prerequisite skills.

Given my school's goal of ensuring college readiness, I firmly believe that students should not be able to move on to algebra 2 until they are ready. I would like to try to institutionalize the main idea behind Soni's proposition. This would mean giving students a test toward the end of algebra 1 . Students who do not pass this test would have to attend summer school. If they can demonstrate mastery, they would move on to the next algebra course their sophomore year. Otherwise, they would repeat algebra 1.

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/I read with interest the rationale for requiring high school students to pass an entrance exam to be allowed to take algebra 2 and, if they fail, to repeat algebra 1 . I teach both courses at a small alternative high school. Most of my students have had little success in mathematics, often because of an inability to complete homework. Many fear the subject. Our school district requires four years of high school mathematics to graduate, and that requirement seems to be more common across the nation. Forcing students to retake algebra 1 would require them to double up on mathematics in their senior year or to attend summer school for credit recovery.

The issue of students not acquiring basic algebra skills the first time around seems to me to be more a teaching issue than something to blame students for. The mathematics textbook publishers have included every early algebra skill possible in algebra 1 textbooks to comply with the myriad state standards that currently exist. Students suffer because many teachers feel compelled to "get through the book," so there is no learning of any skill at a deep level. If students happen to be blessed with natural talent in logical-mathematical intelligence, they can maneuver the flyby of algebra 1 and then succeed in algebra 2 handily. If not, they must struggle through the rest of their high school career with a poor understanding of what I agree is the most important mathematics class for future success. It appears that the Common Core State Standards are addressing this issue by narrowing the curriculum and ensuring that all states are working from the same standards. This approach will allow textbook publishers to tighten their scope and sequence for each high school mathematics course.

We teachers also need to examine our own practice. Are we designing classroom activities that require students to discuss what they need to learn? Are we asking them to synthesize and reflect? Are we ensuring that our curriculum is not a mile wide and an inch deep but instead staying with concepts long enough for deep understanding to occur? I have designed my algebra 1 curriculum as a two-year course to ensure success. Could students be identified early for this type of intervention yet still have time to complete four years of mathematics, including algebra 2 ?

It seems unfair to have students pay the price for adjustments that need to happen on our end. If students are not ready, isn't it our job to help them get ready?

Susan Mick
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/I do not disagree with Soni's assertion that some students need to spend another year in some mathematics classes. I do disagree with his assumption that students who do not master algebra 1 skills by the end of that course will never learn them.

Most recent mathematics textbooks do not assume that students have mastered the material from the previous course. Surveying the algebra 2 textbooks in my office, I note that many start with a chapter reviewing algebra 1. (An older textbook, Foerster's [1st ed., 1980], starts with "The Field Axioms.") In textbooks for later courses, perhaps half the exercises have a glimmer of the current topic, but these exercises are really there for review. Exercises in verifying trigonometry identities are disguised practice in adding fractions, finding differences of squares, and factoring. Witness the engorgement of Lang's A First Course in Calculus, which has ballooned from 250 pages to 700 pages between the first edition (1964) and the fifth (1986), even adding an entire section entitled "Review of Basic Material."

I hope that students leaving my courses have mastered the material, but my realistic expectation is that in their next mathematics course they will vaguely recollect the topics I covered. It is in the next course, when they have to apply those skills, that they will master them. Many academic majors in which students will never use the calculus nonetheless require students to take it, because that's where students master algebra.

I do not expect students coming into my algebra 2 course to have mastered algebra 1 . I have learned not to expect that they can add fractions. Many are only vaguely familiar with addition with carrying and subtraction with borrowing. I have witnessed students using long multiplication to multiply by 10 . Do I send them back to fourth grade?

Students who do not do well in one course will struggle that much harder in the next. For many students, the best approach is to repeat a course.

It is not true, however, that a student who did not master several
algebra 1 topics while in that course will not be able to improve those skills during the next course. The system is designed to support that student because most students are that student.
J. Bradford Burkman bburkman@lsmsa.edu Louisiana School for Math, Science, and the Arts Natchitoches, LA, Mar. 13, 2012

1)Current practice is to require all students to learn mathematics at the same rate. This practice is unreasonable and results in many students concluding that they can't do mathematics. And it doesn't start with algebra 1. I'm in favor of an approach offered by the Khan Academy, where students, at home, view and re-view a video that presents a skill. The students then do "homework" in class, where the teacher and other higher-achieving students help the slower students individually.

Algebra 1 should not be a 25 student, 180-day event; algebra 1 should be the time it takes for a student to achieve at a level that ensures success at the next level. If we believe that all students can learn some particular subset of mathematics that is deemed important, then we need to give them some control over the time that they individually require for its acquisition. And we need to help them become aware of how they best learn.

Adults acquire new knowledge and skills by teaching themselves. When challenged to learn something new, mature learners are armed with a sense of how they best learn. They make a choice about whether to read, listen, or view with available technology. They confirm their newly acquired learning through discussions with others and by helping others. Many of us have recently learned to use a smartphone, $\mathrm{iPad}^{\circledR}$, and SMART Board ${ }^{\mathrm{Tn}}$. Each of us
acquired these new skills by using learning strategies of our choice, with complete control over our rate of learning.

Jack McCabe
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Editor's note: This year, McCabe celebrates his fiftieth year of membership in NCTM.

"I read the letter in Mathematics Teacher about second-year algebra students who would be better off repeating first-year algebra. Please send me a copy of the basic entrance exam that you gave your algebra 2 students.

I would like to share this letter and your exam with my department.

## Chris Lodes

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Lincoln, NE, Mar. 3, 2012

## SID SONI RESPONDS

I did not want my letter to be just another unconstructive "The kids can't factor!" mathematics teacher cliché. A gratifying postscript was receiving the varied responses from other high school teachers who felt that I provided the impetus for taking action at the local level. Others might feel that repeating the entire course presents a false dilemma, as Burkman's response indicates. A different plan of action would be to identify an explicit subset of core exit skills for algebra students. These "must have" skills constantly arise in later courses (e.g., solving linear equations, proportional reasoning). During the year, these skills can be emphasized, reinforced independently, drilled and killed, reassigned as summer work, or retested before promotion to the next level. These additional opportunities would make students' chances of mastering the content and succeeding in second-year algebra that much higher.

Should there be shame in repeating a course if it didn't click the first time? I say no. Complex learning can take hundreds of cycles of repetition to nail down. Mathematics, dance steps, golf swings, musical scales, wine tasting, photogra-phy-none of these is a "teach-once, learn-once" skill. In any nontrivial learning process, we need to step back, practice, reprocess, revisit, synthesize, and sometimes drill and kill. Only when we "own" a stage will we get anything out of the following stage.

The larger implications tied to high school algebra ability are worth noting. Failing to master this coursework
can affect all future mathematics and science coursework and will even limit the types of college majors (and, hence, careers) that students may pursue. High school algebra skills include, among others, interpreting symbolic notation; manipulating expressions, formulas, and variables; solving equations; understanding probability; and graphing in the Cartesian plane. These skills are necessary for success in college courses as well. Statistics, calculus, and economics, which make use of data, formulas, equations, and graphing, require these skills. Academic areas that have an explicit statistics or calculus requirement include psychology, nursing, accounting, economics, social work, business management, marketing, finance, chemistry, biology, computer science, physics, and engineering. Clearly, not mastering algebra skills can close numerous academic and professional doors.

One pace does not fit all, which is why McCabe's response is so vital. My letter was written to address a core limitation of the traditional classroom model (i.e., limits of individual pacing). A bigger picture emerges as the very nature of education and autodidactic learning evolves. How will self-paced online learning coalesce or clash with the existing model of education? Instead of a literal interpretation of "repeat the course," the universal value of this discussion is to underscore the importance of adequate foundational prerequisite skills in any sequential learning process. This principle holds true whether the learning occurs in a classroom, on a computer, or during personal growth and scholarship.

