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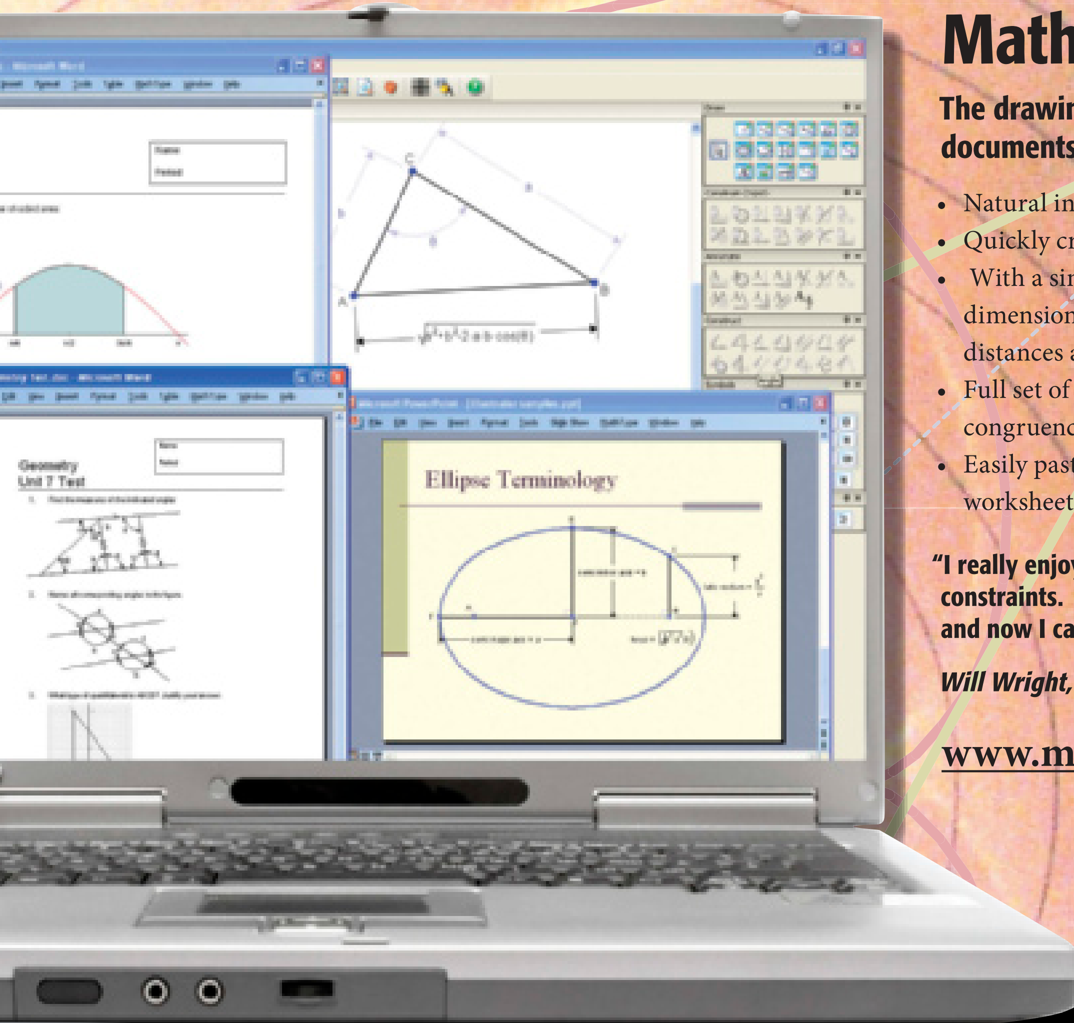

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$$Y=\frac{a^2\cdot r\cdot(3\cdot\sin(t)-\sin(3\cdot t))}{2\cdot\left(2\cdot a^2+r^2+3\cdot a\cdot r\cdot\cos(t)\right)}$$

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Ariel Chen, Junior, Westview High School

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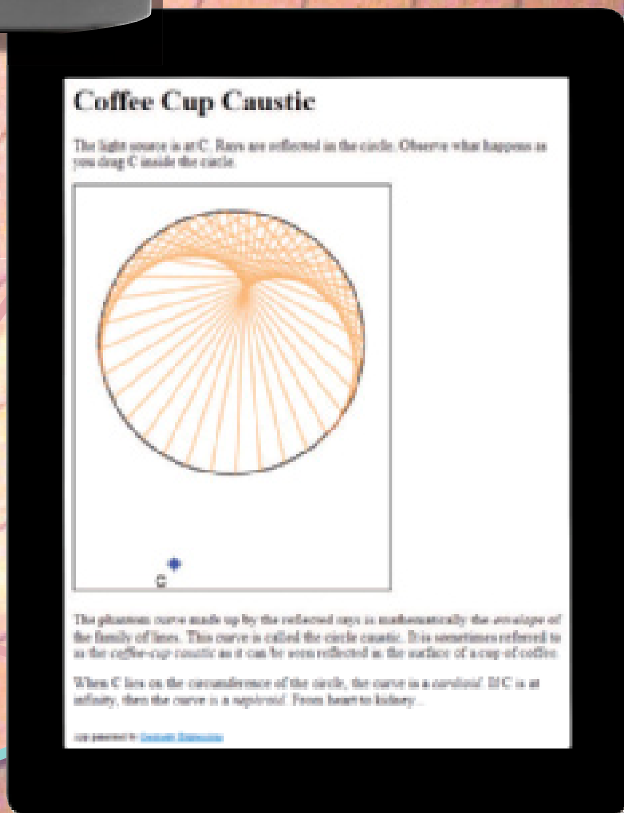

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READER reflections

TWO MORE SOLUTIONS FOR A PROBLEM WITH MANY

In "One Problem, Six (or More!) Solutions" in the September 2011 *Mathematics Teacher* (Delving Deeper, vol. 105, no. 2, pp. 150–55), the authors provided a total of eight solutions: six in the article itself and two additional ones on the *MT* website. I wish to suggest two more solutions to this problem.

Solution 1

The equation of the tangent line passing through point P (9, –2) with slope m can be expressed as $mx - y - 9m - 2 = 0$ (see **fig. 1 [Tu]**). The distance from the center Q (2, –1) to the tangent line, PT or PT', is equal to the radius of the circle. Hence,

$$\frac{|2m+1-9m-2|}{\sqrt{m^2+1}} = 5.$$

Simplifying yields a quadratic equation with respect to m: $12m^2 + 7m - 12 = 0$. This equation produces $m = -4/3$ and $m = 3/4$. Therefore, the equations of the tangent line are

$$4x + 3y = 30 \text{ and } 3x - 4y = 35.$$

We appreciate the interest and value the views of those who write. Readers commenting on articles are encouraged to send copies of their correspondence to the authors. For publication: All letters for publication are acknowledged, but because of the large number submitted, we do not send letters of acceptance or rejection. Letters to be considered for publication should be in MS Word document format and sent to **mt@nctm.org**. Letters should not exceed 250 words and are subject to abridgment. At the end of the letter include your name and affiliation, if any, including e-mail address, per the style of the section.

Solution 2

Note that if T (x_0, y_0) is the point of tangency on the circle $(x - a)^2 + (y - b)^2 = r^2$, then the equation of the tangent line at T (x_0, y_0) is $(x_0 - a)(x - a) + (y_0 - b)(y - b) = r^2$. Assume that T (x_0, y_0) is the point of tangency. Then the equation of the tangent line is

$$(x_0 - 2)(x - 2) + (y_0 + 1)(y + 1) = 25 \qquad (1)$$

We set $u = x_0 - 2$ and $v = y_0 + 1$, and since this tangent line passes through P (9, –2), equation (1) becomes

$$7u - v = 25. \qquad (2)$$

In addition, since the point T (x_0, y_0) is on the circle, it satisfies $(x_0 - 2)^2 + (y_0 + 1)^2 = 25$, or

$$u^2 + v^2 = 25. \qquad (3)$$

Solving the system consisting of equations (2) and (3), we find that $u = 3$ or 4 and $v = -4$ or 3.

Therefore, the equations of the tangent lines are $3(x - 2) - 4(y + 1) = 25$ and $4(x - 2) + 3(y + 1) = 25$, or $4x + 3y = 30$ and $3x - 4y = 35$.

The ICTM (Illinois) Regional 2005 Division AA, Junior-Senior 8 Person Team competition includes a similar question:

> A circle has equation $(x - 7)^2 + (y - 1)^2 = 10$. Find the slope of the tangent to this circle if the tangent passes through (10, 2).

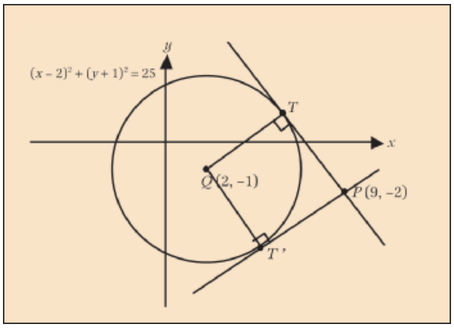

Fig. 1 (Tu)

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Oswego, IL, Sept. 30, 2011

A KINESTHETIC APPROACH TO HORIZONTAL SHIFT

I enjoyed reading the article "Algebra Aerobics" by Julie Barnes and Kathy Jaqua (*MT* September 2011, vol. 105, no. 2, pp. 96–101). As the authors state, $f(x - 1)$ does not resonate intuitively as a shift to the right. I would like to suggest an activity that can facilitate the understanding of this horizontal shift and that uses the coordinate system described in the article.

Using the classroom floor as the coordinate plane, have five students stand with their backs to the x-axis on the following coordinates: (1, 0), (2, 1), (3, 2), (4, 3), (5, 4). These points represent the function $f(x)$. Another student attaches a red tape through these points. The five students step 1 unit to the right. They now stand on (2, 0), (3, 1), (4, 2), (5, 3), and (6, 4) These points define a new function called $g(x)$. Asked about the change that occurred, the students observe that the y-coordinates did not change but that the x-coordinates did. A student attaches a blue tape through these five points. The five students step off the points, and all the students observe the two functions: the original red one and the "new" blue one

CONTINUE the discussion

Mathematics Teacher invites readers to respond to the following letter. Responses will be published in the August 2012 issue of the journal. To be included, responses must reach us by April 1, 2012. Send responses to mt@nctm.org.

THE CASE FOR REPEATING ALGEBRA 1

At midyear in my algebra 2 course, I distributed my students' grades, which ranged from the 40s to the 90s. I felt that the failing students had simply not mastered the prerequisite algebra skills required to perform in this course. Although this is a painfully obvious connection, I administered an informal diagnostic to see just how closely my students' first-year algebra skills correlated with their second-year algebra performance. I deliberately chose prosaic algebra topics that should require no preparation or memorization but should be automatic skills for anyone who has taken the course—plotting a Cartesian point; solving a two-step equation; understanding order of operations, the distributive property, simple probability; and so on.

The correlation between the students' scores on the algebra skills diagnostic and their algebra 2 midyear average was $r = .75$ (see **fig. 1 [Soni]**). Further, the bottom 20th percentile (those students scoring 44% or less) correlated to a 100% fail rate in the algebra 2 course at midyear. This was strong evidence that this subset of the class simply did not have the prerequisite algebra skills to pass algebra 2. What steps can be taken to avoid this situation in the future?

Course averages are sometimes padded with homework points, participation points, and scores from retaken tests. As a result, some students may technically pass a first course in algebra yet not have a firm grasp of the subject. To circumvent this predicament, courses with axiomatic prerequisite skills should require students to pass a basic entrance exam; the one I administered consisted of fifteen questions and took less than one period.

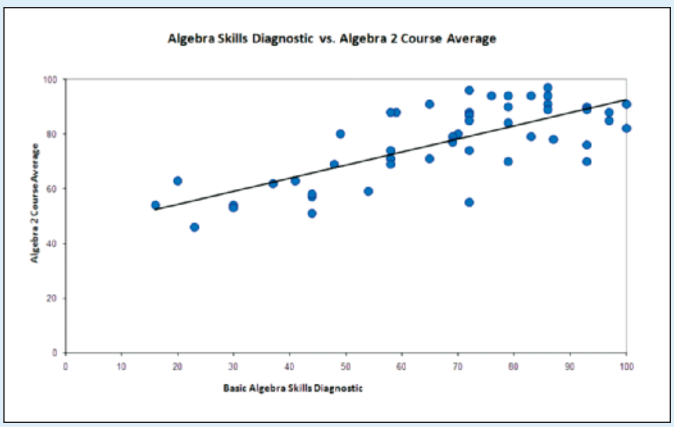

Fig. 1 (Soni)

The ostensible choice is either to promote the student into a class he or she probably cannot pass or give him or her another chance to master the basics under different circumstances—having a different teacher, being one year older, seeing the material again, and so on. For students who cannot demonstrate algebra skills, I believe that repeating first-year algebra is the best recourse. Weak algebra foundations will have substantial downstream effects on subsequent mathematics and science courses. Ergo, the exit requirements of algebra 1 should be the most stringent of any high school mathematics course.

One rationale for promoting borderline students is to ensure that they complete four years of high school mathematics, thus bolstering their college applications. This logic seems flawed for three reasons:

1. What mathematics can a student learn without possessing basic ninth-grade algebra skills?
2. Weak algebra skills will hamper performance in geometry and second-year algebra.
3. Marginal algebra skills will prevent students from doing well on college entrance exams.

Weak students, despite having taken four years of mathematics, will not likely qualify for the caliber of college that encourages doing so.

Further, at many colleges, particularly less competitive ones, incoming students are given a placement exam that is heavily skewed toward measuring their algebraic skills. They will take this exam after a two-year hiatus from this material and several summer vacations. If I had to wager on the outcome of this exam, I would predict that this cohort will be retaking algebra in college, thus begging the question: Are these students better off repeating algebra in grade 10 or in grade 13?

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Westchester County, NY, Feb. 24, 2011

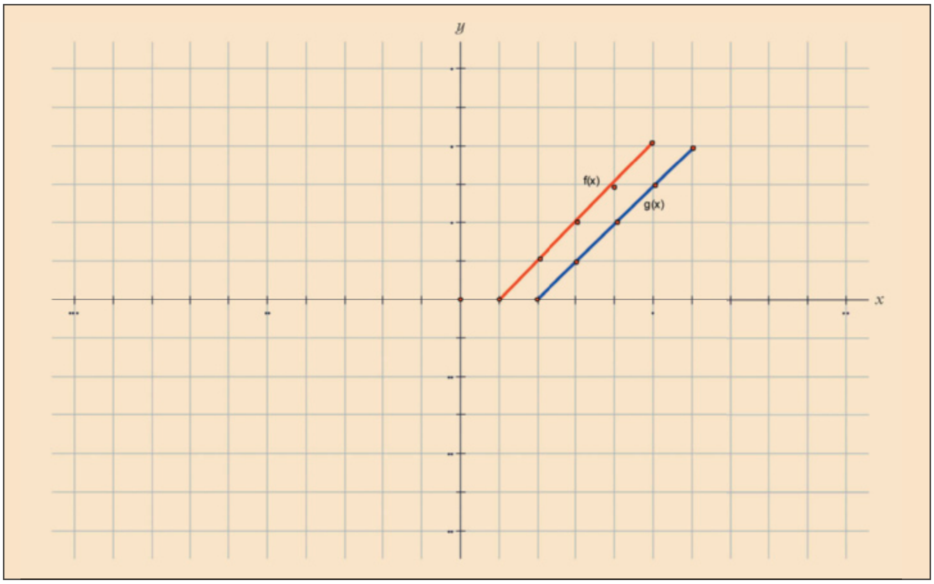

Fig. 1 (Touval)

(see **fig. 1 [Touval]**).

The teacher then asks: "You created a new function $g(x)$, colored here in blue. If x is the x in the new function $g(x)$, how do we form an equation that relates $g(x)$ and $f(x)$?" Students will figure out that by subtracting 1 from x, they can find $f(x - 1)$, which is equal to $g(x)$. Therefore, $f(x - 1) = g(x)$.

Conclusion: Yes, we shifted right from the function $f(x)$. But to find the y-value of x of the new function, we look for the y-value of $x - 1$.

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PROBLEMS 2, 15, 23, AND 26, SEPTEMBER 2011 CALENDAR

Following are alternate solutions to selected problems from the September 2011 Calendar (*MT* August 2011, vol. 105, no. 1, pp. 40–46).

Problem 2

The leftmost term of the nth row is given by $n^2 - n + 1$. The proof is similar to that shown in the solution to problem 10. The first term of the 18th row can then be found as $a(1) = 18^2 - 18 + 1 = 307$. This row has 18 terms in ascending order, and the median is given by the average of the 9th and 10th terms. Since $a(9) = a(1) + (9 - 1)(2) = 323$ and $a(10) = 325$, the median is $(323 + 325)/2 = 324$.

Problem 15

Once we have found the intersection points of the three lines, we can solve this problem using determinants, where the area is half the determinant of the 3×3 matrix:

$$\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}$$

In this situation, we have

$$Area = \frac{1}{2} \bullet \begin{vmatrix} 0 & 0 & 1 \\ 1 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix}$$

$$= \left| \frac{1(2-8) - 1(0-0) + 1(0-0)}{2} \right|$$

$$= 3.$$

Problem 23

The chart shown in **figure 1 (Sriskandarajah)** helps clarify problem 23. The total number of pennies is $P = P/4 + 4P/5 - 3$. Solving this equation yields $P = 60$.

	Number of Heads	Number of Tails
Initially	$P/5$	$4P/5$
Later	$P/4$	$4P/5 - 3$

Fig. 1 (Sriskandarajah)

Problem 26

The total number of terms in the triangular array with n rows is $1 + 2 + 3 + \cdots + n = n(1 + n)/2$. From problem 10, we know that the rightmost (or last) term of this triangular array is $n^2 + n - 1$. But the row sums are $R(1) = 1 = 1^3$; $R(2) = 3 + 5 = 8 = 2^3$; $R(3) = 7 + 9 + 11 = 27 = 3^3$; ...; $R(n) = n^3$.

Hence, the sum of the first n cubes, $1^3 + 2^3 + 3^3 + \cdots + n^3$, is the sum of all the terms in this triangular array of odd integers $1 + 3 + 5 + \cdots + (n^2 + n - 1)$, which is an arithmetic series with common difference of 2. The sum of this sequence is

$$\frac{n(1+n)}{2} \bullet \frac{1+(n^2+n-1)}{2} = \left(\frac{n(n+1)}{2}\right)^2.$$

This result is in fact $(1 + 2 + 3 + \cdots + n)^2$.

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WHEN IS A PARABOLA NOT A PARABOLA?

The graph of $y = x^2$ is shown in **figure 1 (Foster)**. Or is it? Does anything strike you as not quite right about this image?

If we superimpose a genuine parabola (dashed), we see the difference (see **fig. 2 [Foster]**). The curve in **figure 1** matches at the origin and at the two endpoints of our parabolic segment but is too "pointy" in between. These "pointy parabolas" are everywhere—in mathematics textbooks, on the Internet, even on examination papers. Once you start looking for these

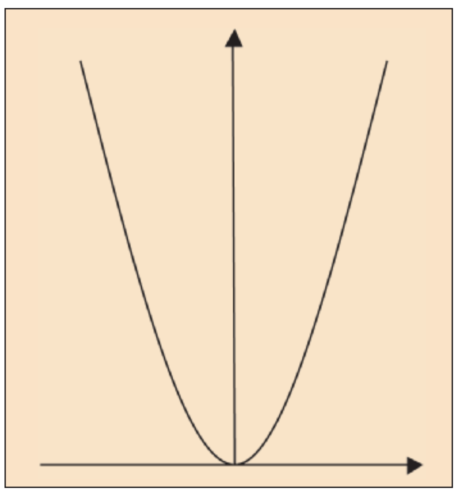

Fig. 1 (Foster)

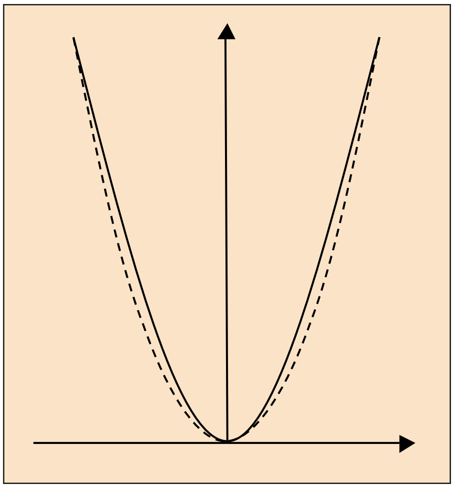

Fig. 2 (Foster)

pointy curves, you see them all over the place. The same problem occurs with cubics (see **fig. 3 [Foster]**) and with most other mathematical curves.

These pointy curves come from computer drawing software, in which you click on a finite number of points (called "control points") and the computer creates a smooth curve linking them. In

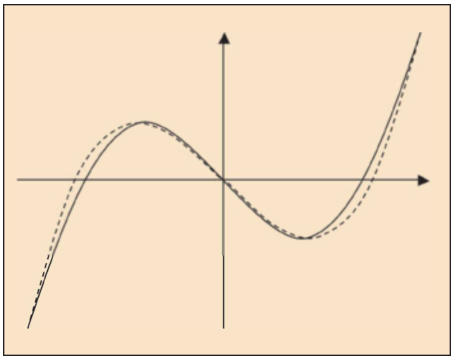

Fig. 3 (Foster)

time gone by, the software used Bézier curves, and the curve did not necessarily pass through the points you clicked on (except for the first and the last). With three control points, these curves (quadratic Bézier curves) would produce a parabolic segment, so you could draw your $y = x^2$ without any problem. But it could be difficult to get exactly the curve that you wanted by dragging control points, which were often not on the curve. So more modern software tends to use "'curve through points" algorithms, producing such things as cubic splines, which lead to a smooth curve passing through all the control points, the position of the *n*th point determining the shape of the curve from the $(n - 2)$th point onward.

You may take the view that since these drawings are just mathematical sketches, it doesn't matter whether they are accurate; no one is supposed to be measuring them. However, by repeatedly using these inaccurate curves, we are in danger of systematically distorting some of the canonical images of elementary mathematics—standard curves that students should recognize as "friends."

This is a little plea for teachers to take a bit more trouble and draw curves using some of the excellent graph-drawing software now available. We shouldn't fob off "pointy" pseudo-parabolas on our students.

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Coventry, UK, Feb. 22, 2011