

PROVE: that  $\sqrt{2}$  is irrational.

ie prove there is no <sup>repeating</sup> pattern in the decimal.

Maybe it's just real long, and you didn't notice it, stupid human.

Assume:  $\sqrt{2}$  is rational.

Then  $\sqrt{2} = \frac{a}{b}$  [definition of rational] AND  $\frac{a}{b}$  is simplified to lowest terms

It follows that  $2 = \frac{a^2}{b^2}$  (squared both sides)

Note: that an odd \* odd = odd  
even \* even = even } always

if  $2 = \frac{a^2}{b^2}$  then multiply by  $b^2$  to get:  $2b^2 = a^2$   
(both sides)

$a^2$  must be even, since it is equal to 2 times something ( $b^2$ )

if  $a^2$  is even, then  $a$  is even. (no way odd \* odd = even)

if  $a$  is even, then by definition,  $a = 2K$  (any even is 2 \* some #)

go back to original claim:  $2 = \frac{a^2}{b^2} \Rightarrow 2 = \frac{(2K)^2}{b^2} \Rightarrow 2 = \frac{4K^2}{b^2}$   
(Substitute)

$$2b^2 = 4K^2$$

$b^2 = 2K^2 \rightarrow$  means  $b^2$  is even, so  $b$  is even

WAIT! if  $a$  and  $b$  are both even  $\rightarrow \frac{a}{b}$  was NOT simplified!  
 $\therefore$  Contradiction:  $\sqrt{2}$  is NOT Rational!